Beam Instability and Microbunching due to Coherent Synchrotron Radiation

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Introduction

- A relativistic electron beam moving on a circular orbit in free space can radiate coherently at the wavelengths that exceed the length of the bunch.
- Coherent radiation at shorter wavelengths can result from density fluctuations in the beam with characteristic length much shorter than the bunch length.
- If the radiation reaction force drives the growth of the initial fluctuation, one can expect an instability which leads to micro-bunching of the beam and increased coherent radiation at short wavelengths.

Mechanism of the instability

Let us assume a small initial sinusoidal density perturbation on the beam, $\delta n = \epsilon \sin kz$

- Due to the CSR wake, δn induces energy modulation in the beam δE_1
- Momentum compaction of the ring translates δE_1 into δn. Under certain conditions, the final δn is greater than the initial one.
- Energy spread introduces Landau damping and stabilizes short wavelengths.
- Wall shielding of CSR and finite length of the bunch limits the instability at large wavelength.
- Transverse beam emittance mixes the particle over the wavelength and may have a stabilizing effect on the instability

Theory (Heifets, Stupakov, 2002)

Since we are interested in the wavelength much shorter than the bunch length, consider a *coasting* beam moving in a circular orbit of radius R in free space.

$\rho(\delta, z, s)$ — longitudinal distribution function, $dN = dz \int \rho(\delta, z, s) d\delta$.

Vlasov equation (neglect damping and quantum diffusion)

$$\frac{\partial \rho}{\partial s} - \eta\delta \frac{\partial \rho}{\partial z} - \frac{r_0}{\gamma} \frac{\partial \rho}{\partial \delta} \int_{-\infty}^{\infty} dz' d\delta' W(z - z') \rho(\delta', z', s) = 0$$

$\delta = \Delta E/E$

$s = ct$

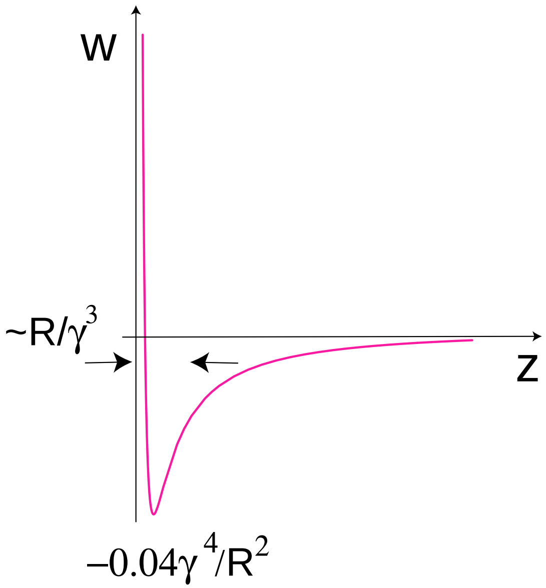
η – slip factor

r_0 – classical electron radius

$W(z - z')$ – wake function (per unit length of the path).

CSR wake

A relativistic particle moving in vacuum in a circular orbit of radius R, in steady state, generates a CSR wake (per unit length of path) (Murphy et al., 1995; Derbenev et al. 1995)



$$W(z) \approx -\frac{E_{\parallel}}{q} = -\frac{2}{3^{4/3}R^{1/3}z^{4/3}}$$

This wake neglects transient effects at the entrance to and exit from the magnet. If is valid if (λ – wavelength)

$$l_{\text{magnet}} > \lambda^{1/3}R^{2/3}$$

- For $R \approx 30$ m and $\gamma \approx 10^4$, $R/\gamma^3 = 3 \cdot 10^{-11}$ m—negligibly small
- Shielding effects become important at the distance $z \gtrsim \frac{R^{1/2}}{a^{3/2}}$, a – gap between walls

Neglect the shielding effect, and assume a steady-state wake

$$W(z) = \frac{2}{(3R^2)^{1/3}} \frac{\partial}{\partial z} \frac{1}{z^{1/3}} \quad \text{for } z > 0,$$

and $W(z) = 0$ for $z \leq 0$. The radiation wakefield is localized in front of the moving charge.

Impedance

$$Z(k) = \frac{1}{c} \int_0^\infty dz W(z) e^{-ikz} = iA \frac{k^{1/3}}{cR^{2/3}}.$$

The complex factor A is

$$A = 3^{-1/3} \Gamma\left(\frac{2}{3}\right) \left(\sqrt{3}i - 1\right) = 1.63i - 0.94$$

For a perturbation $\rho_1 \propto e^{-i\omega s/c + ikz}$, the dispersion relation gives dependence ω vs k

$$1 = -\frac{ir_0 c^2 Z(k)}{\gamma} \int \frac{d\delta\, (d\rho_0/d\delta)}{\omega + ck\eta\delta}$$

This is a standard formula for a coasting beam instability.

If bends do not fill the whole ring, we introduce a weighting factor in Z

$$Z \to Z \frac{R}{\langle R \rangle}$$

where $\langle R \rangle = C/2\pi$.
For a Gaussian distribution function, $\rho_0 = n_b (2\pi)^{-1/2} \exp(-\delta^2/2\delta_0^2)$

$$\frac{(kR)^{2/3}}{\Lambda} = -\frac{A}{\sqrt{2\pi}} \int_{-\infty}^{\infty} \frac{dp\, p\, e^{-p^2/2}}{\Omega + p}$$

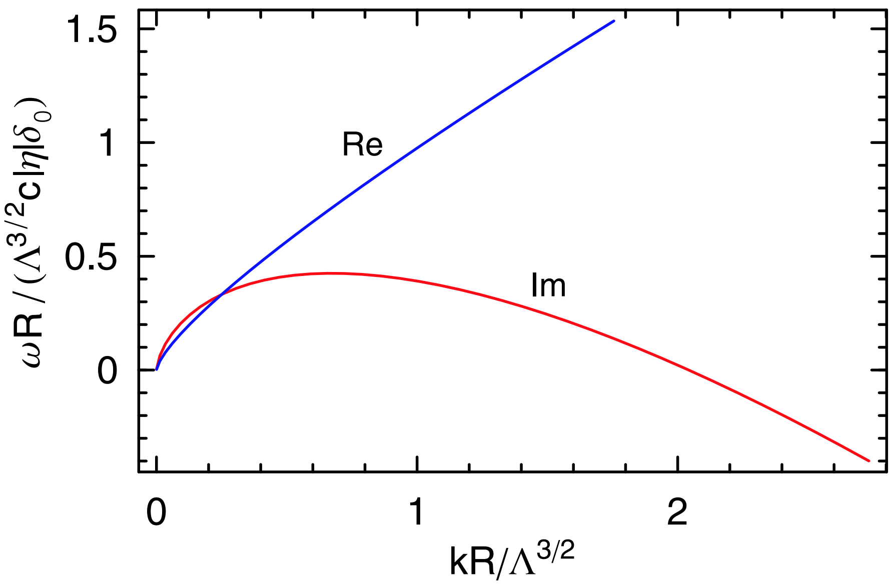
where $\Omega = \omega / ck\eta\delta_0$,

$$\Lambda = \frac{n_b r_0}{|\eta| \gamma \delta_0^2} \frac{R}{\langle R \rangle}$$

Note

$$n_b r_0 = \frac{I}{17 \text{ kA}}$$

ω versus k for positive η



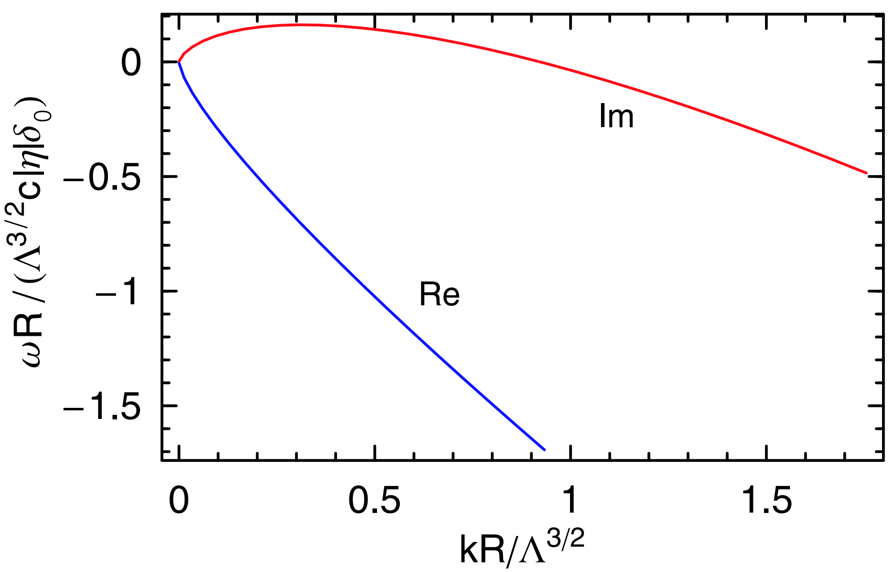
The beam is unstable for such wavelength that

$$kR < 2.0\Lambda^{3/2}.$$

$$\Lambda \propto \frac{I}{\delta_0^2}$$

The maximum growth rate is reached at $kR = 0.68\Lambda^{3/2}$ and is equal to $\mathrm{Im}\omega_{\mathrm{max}} = 0.43\Lambda^{3/2} c\eta\delta_0/R$.

ω versus k for negative η



The beam is unstable for such wavelength that

$$kR < 0.92\Lambda^{3/2}.$$

The maximum growth rate is reached at $kR = 0.32\Lambda^{3/2}$ and is equal to $\mathrm{Im}\omega_{\max} = 0.16\Lambda^{3/2}c|\eta|\delta_0/R$.

Limit of the cold beam, $k \ll \Lambda^{3/2}/R$,

$$\operatorname{Im}\omega = 1.2c\left(\frac{r_0 k^{4/3} n_b \eta}{\gamma R^{2/3}}\right)^{1/2}$$

No energy spread δ_0 in this formula.

We neglected the synchrotron damping γ_d due to incoherent radiation. The effective growth rate of the instability $\sim \operatorname{Im}\omega - \gamma_d$ (see details in Heifets and Stupakov, SLAC-PUB-8803).

If we want to see the instability ...

1. Bunched Beam. For a bunched beam of length σ_z the coasting-beam approximation can be applied if $k\sigma_z \gg 1$,

$$\sigma_z \gtrsim 0.5\frac{R}{\Lambda^{3/2}}$$

2. Shielding. Finite aperture a of the beam pipe — CSR is suppressed due to the shielding effect at

$$kR \lesssim \left(\frac{\pi R}{2a}\right)^{3/2}$$

Hence the instability can only develop for such values of k that $2.0\Lambda^{3/2} > kR \gtrsim (\pi R/2a)^{3/2}$.

$$\frac{R}{a} \lesssim \Lambda.$$

Numerical Estimates for LER, ALS and VUV rings

Accelerator	LER PEP-II	ALS	VUV NSLS
E (GeV)	3.1	1.5	0.81
η	$1.31 \cdot 10^{-3}$	$1.41 \cdot 10^{-3}$	$2.35 \cdot 10^{-2}$
δ_0	$8.1 \cdot 10^{-4}$	$7.1 \cdot 10^{-4}$	$5.0 \cdot 10^{-4}$
$\langle R \rangle$ (m)	350	31.3	8.11
R (m)	13.7	4	1.91
a (cm)	2	1	2.1
I_b (mA)	2	7.6 (30)	400
σ_z (cm)	1	0.7	4.7
Λ	7	$306\ (1.2 \cdot 10^3)$	250
R/a	550	400	90
$R/2\Lambda^{3/2}$ (cm)	1.0	$0.037(4.7 \cdot 10^{-5})$	0.025

Discussion—formation length and retardation

We neglected retardation in the Vlasov equation. This is valid if the wake formation time is much smaller than the inverse growth rate of the instability:

$$t_{\text{form}} \sim \frac{R}{c}\frac{1}{(kR)^{1/3}} \ll \frac{1}{\operatorname{Im}\omega}.$$

Using for the characteristic wavenumber and frequency of the instability $kR \sim \Lambda^{3/2}$ and $\operatorname{Im}\omega \sim \Lambda^{3/2}c|\eta|\delta_0/R$ yields the condition of applicability of the theory

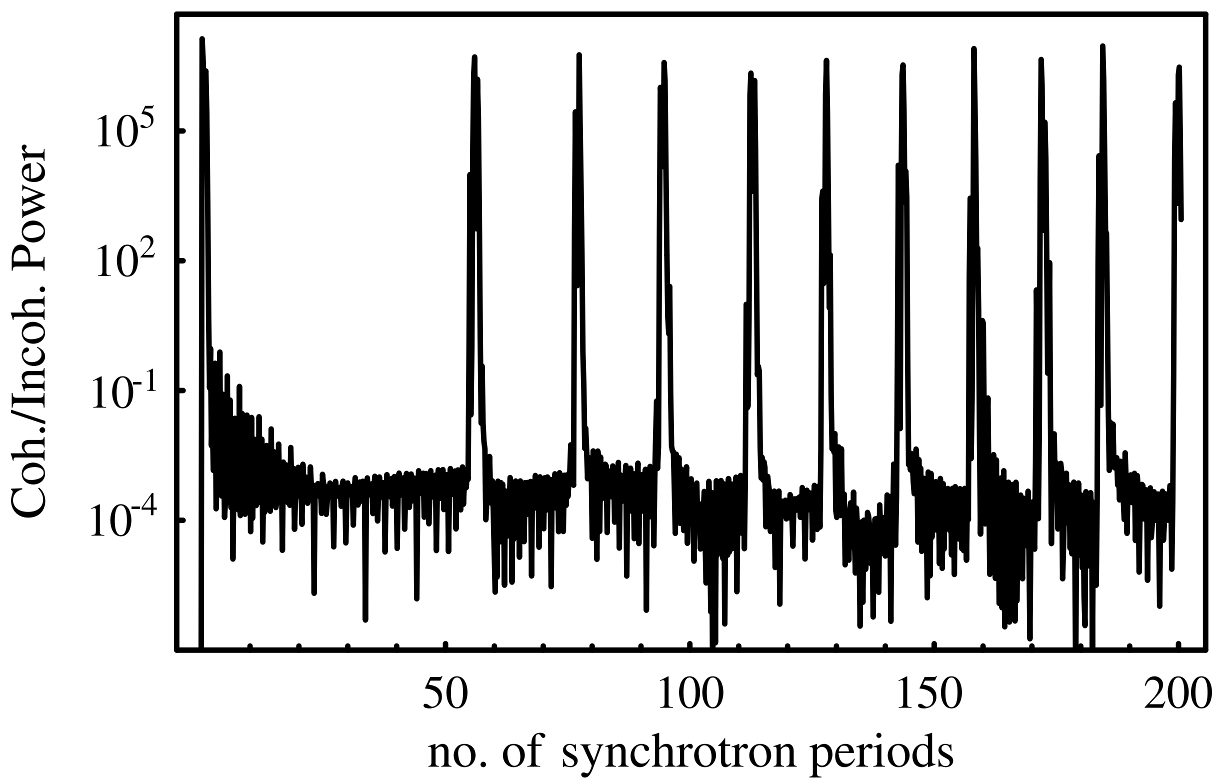
$$\frac{n_0 r_0}{\delta_0 \gamma} \ll 1.$$

Discussion—finite transverse size of the bunch

The CSR wake is not applicable for very short wavelength. This wake was derived for a bunch that is infinitely thin in the transverse direction and assumes that all particles in the cross section of the bunch radiate coherently. However, the transverse coherence length $l_\perp \sim k^{-2/3} R^{1/3}$ decreases with the wavelength and for very large values of k becomes smaller than the transverse dimension of the beam.

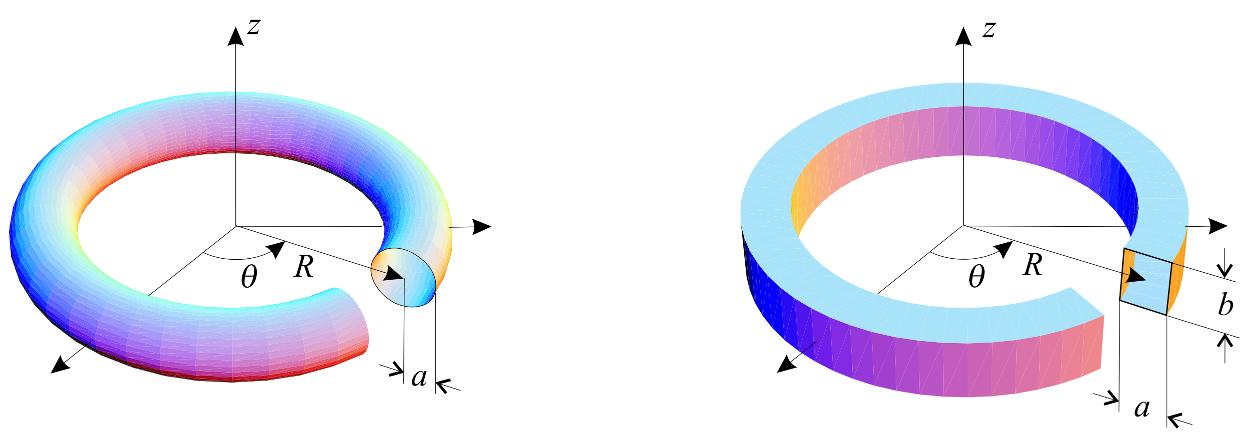
Nonlinear Regime

Not much can be done analytically. See paper by M. Venturini and R. Warnock, SLAC-PUB-9505.



Shielding and discrete modes

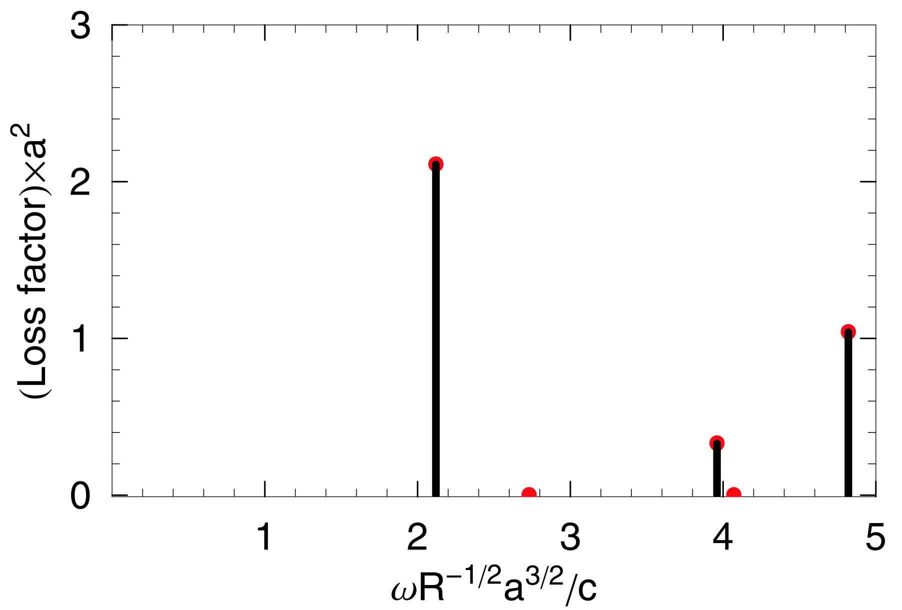
Near the shielding threshold, $\lambda \sim a^{3/2}/R^{1/2}$, the vacuum CSR impedance is not applicable. In the model of a toroidal waveguide with perfectly conduction walls and circular orbit, there are discrete synchronous modes that interact with the beam (B. Warnock&P. Morton, K.-Y. Ng, et al.). We did a new analysis of the shielded CSR impedance (G. Stupakov and I. Kotelnikov, SLAC-PUB-9553), which deals with arbitrary shape of the toroid cross section.



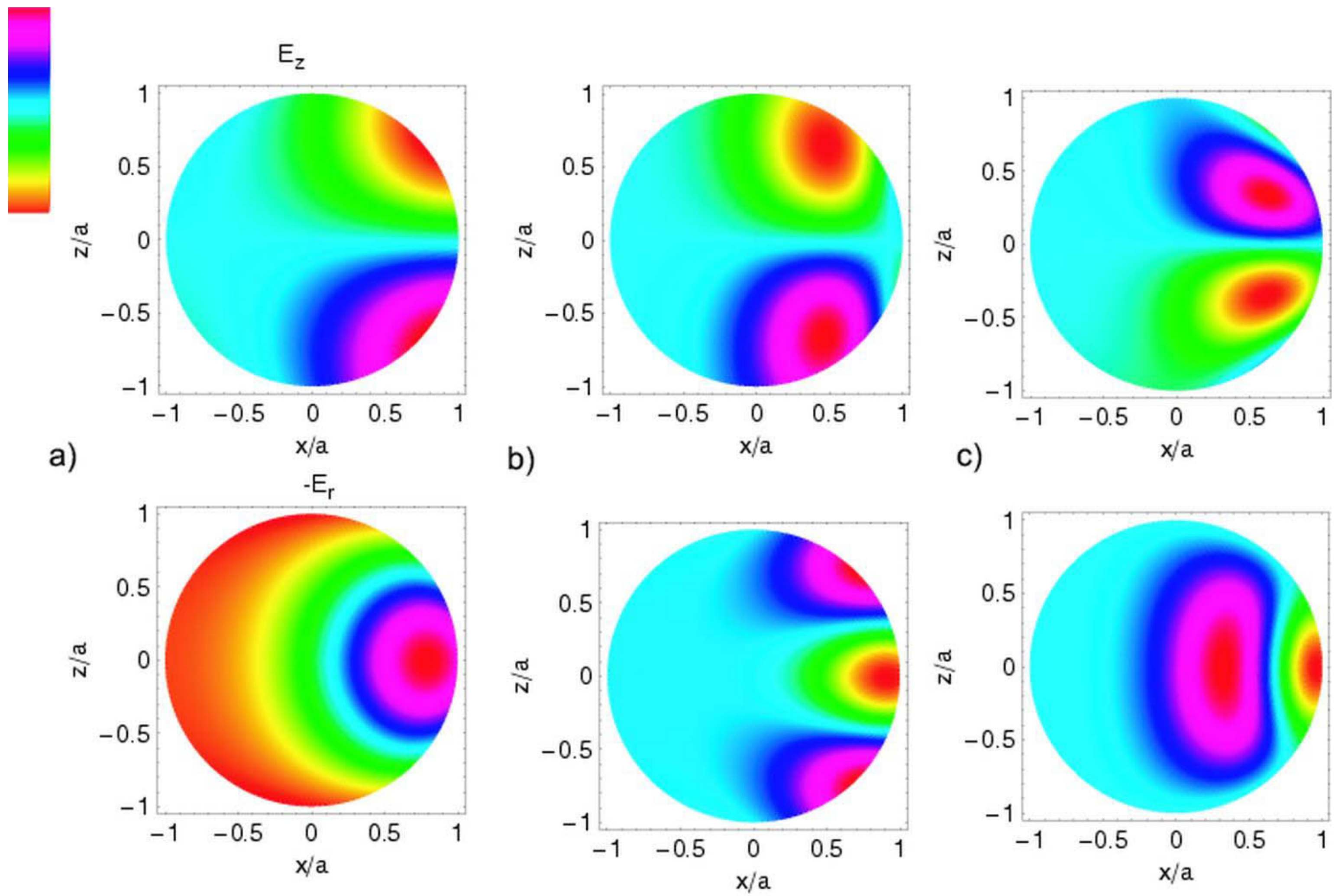
Each mode is characterized by the loss factor (per unit length)

$$w(z) = 2\kappa \cos\left(\frac{\omega}{c} z\right)$$

Fig. below—loss factors for a round toroid.

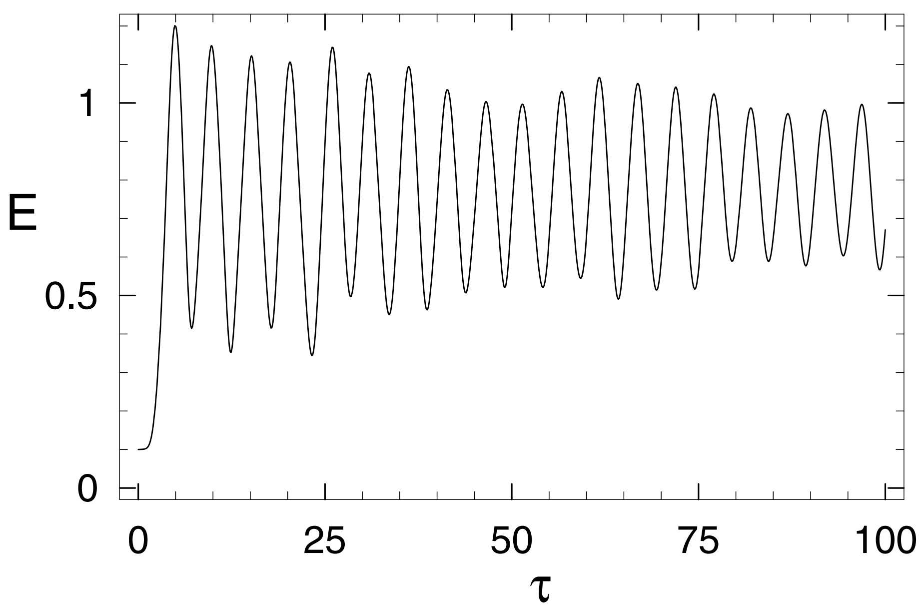


For $R = 4$ m and $a = 1$ cm, the unit of frequency $(\omega/2\pi)$ on the plot is 95 GHz, the unit of loss factor is 90 V/pC/m.


E_z
-E_r
a)
b)
c)
x/a
z/a
−1 −0.5 0 0.5 1

CSR instability should be treated as interaction with single modes, not a continuous spectrum (talk by S. Heifets). The theory is similar to 1D SASE FEL instability (the equations in scaled variables are identical)

$$\Gamma \sim I^{1/3}$$

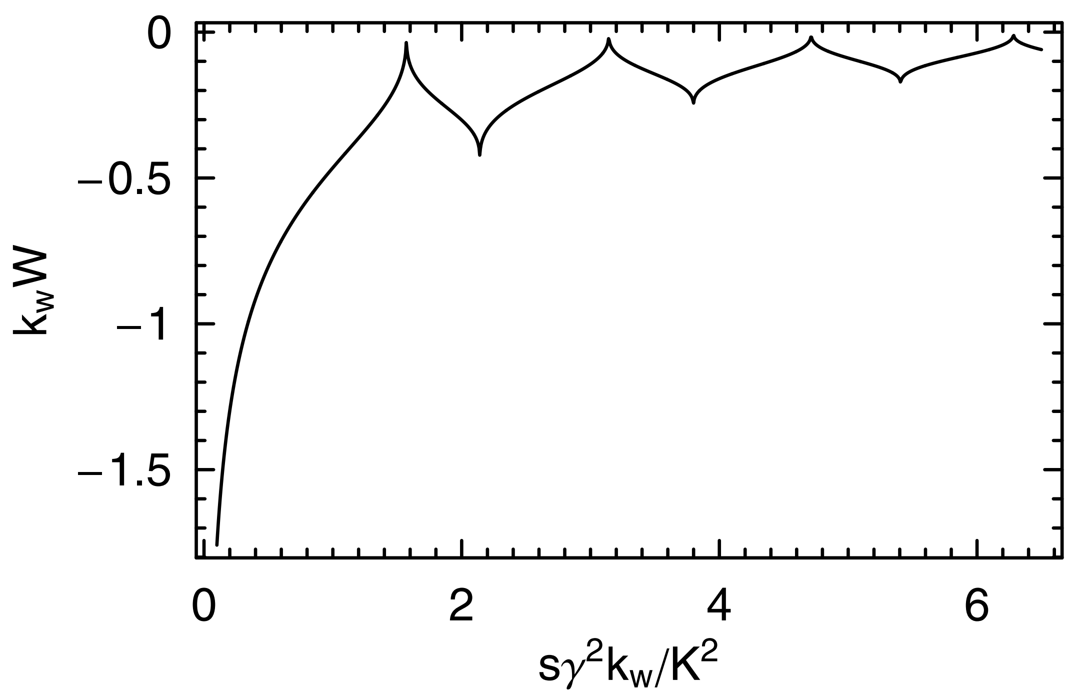


The promise of this theory is a possibility of quasi-continuous radiation (no bursts).

CSR instability in wiggler

Sometime rings have wigglers. Example: NLC damping ring—$B = 2.15$ T, $\lambda_w = 27$ cm, $L_w = 46$ m, $C = 300$ m. How does wiggler effect the CSR instability? We need the CSR wake for the undulator.

Wiggler wake potential in the limit $K^2/2 \gg 1$.



Talk by J. Wu.

Conclusion

1. The theory of CSR instability in the ring is under rapid development. Areas of research include:

- Single mode CSR instability—both linear and nonlinear regimes
- Instability in a ring with wigglers
- Shielding effect, arbitrary cross section of the vacuum chamber
- Computer simulation—finite bunch length, shielding, nonlinear effects, radiation damping and diffusion

2. I see the challenge in searching for regimes where the beam is unstable, but the instability saturates into a steady state with a large CSR radiation.